
2010–2011

MATHCOUNTS[®]

School Handbook:

Volume II

Contains 200 creative math problems
that meet NCTM standards for grades 6-8.

For questions about your local MATHCOUNTS program,
please contact your chapter (local) coordinator. Coordinator contact information
is available through the Find a Coordinator option of the Competition Program
link on www.mathcounts.org.


© 2010 **MATHCOUNTS Foundation**
1420 King Street, Alexandria, VA 22314
703-299-9006
info@mathcounts.org www.mathcounts.org

Unauthorized reproduction of the contents of this publication is a violation of applicable laws.
Materials may be duplicated for use by U.S. schools.

MATHCOUNTS[®] and Mathlete[®] are registered trademarks of the MATHCOUNTS Foundation.



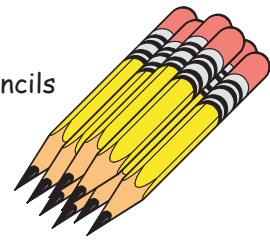
Warm-Up 7

- _____ Melinda chooses a three-digit positive integer, subtracts it from 3000, and triples the result. What is the largest integer Melinda can get?
- _____ Each term of a sequence, after the first term, is three less than the square of the preceding term. If the first term of the sequence is 2, what is the 2011th term?
- _____ hours Susan reads at a rate of 240 words per minute. How many hours will it take her to read a 480-page book that averages 600 words per page?
- _____ A fair coin is to be flipped 5 times. What is the probability that the result will not be 5 heads in a row? Express your answer as a common fraction.
- _____ degrees What is the degree measure of the only angle that is congruent to its complement?
- _____ If $a + 2b = 11$ and $a - b = -4$, what is the value of $4a - b$?
- _____ ₂ What is the value of 1011_3 expressed as a numeral in base 2?
- _____ integers From the set of digits $\{1, 2, 3, 4, 5\}$ three-digit, positive integers are formed, none of which has repeating digits. How many such odd integers can be made?
- _____ % Triangle ABC has vertices with coordinates $(1, 0)$, $(5, 0)$, and $(3, 7)$. Kadim accidentally switched the x - and y -coordinates of every single vertex when calculating the area of the triangle. What is the percent of change between the area of the original triangle ABC and the area of the new triangle Kadim made?
- _____ times  Eight students met for the first time at a MATHCOUNTS competition. Every one of them exchanged cell phone numbers with each of the other seven students, and they all entered the phone numbers into their phones. How many times were cell phone numbers entered into phones among the group of 8 students?





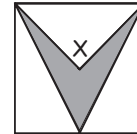
Warm-Up 8



1. \$ _____ At the school store pencils and pens have different prices. Six pencils and four pens cost \$4.30. However, four pencils and six pens cost \$5.20. What is the cost of five pencils and five pens?

2. _____ What term is the smallest five-digit palindrome in the arithmetic sequence 2, 7, 12, 17, ... ?

3. _____ sq yds A chevron is inscribed in a square of side 12 yards where point X is at the center of the square and the upper corners of the chevron touch the upper corners of the square. What is the area of the chevron?



4. _____ On a number line, what is the positive difference between the two numbers that are the trisection points of the line segment with endpoints at $\frac{1}{8}$ and $\frac{3}{4}$? Express your answer as a common fraction.

5. _____ minutes Manny can mow his one-acre yard in 1.5 hours on his riding lawn mower. Timmy takes five times as long to mow Manny's yard with his push mower. How many minutes will it take them working together?

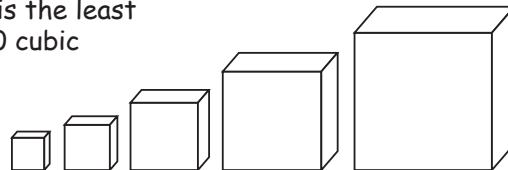


6. _____ What is the units digit of the product $(3^{75})(2^{113})$?

7. _____ What is the value of the expression $\frac{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}}{\frac{1}{2} + \frac{1}{5} + \frac{1}{6}}$? Express your answer as a common fraction.

8. _____ The product of three different positive integers is 2010. What is the maximum possible sum of the three integers?

9. _____ boxes You have a supply of boxes of volumes 1, 3, 9, 27 and 81 cubic meters. Given that the boxes must be filled completely, what is the least number of boxes that will hold exactly 300 cubic meters of sand?



10. _____ If $F(n) = 3n - 5$, $G(n) = n^2 + 3n - 2$ and $H(n) = 15 - 0.3n$, what is the value of $\frac{F(4) \times H(-10)}{G(-3)}$?



Workout 4

1. _____ cm^3 When water freezes, its volume is increased by one-eleventh. In other words, the volume of ice equals the sum of the volume of the original amount of water and the product of one-eleventh and the volume of the water. If 979 cubic centimeters of water is to be frozen, what will be the volume of the ice that will be formed?

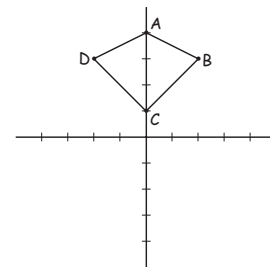
2. _____ meters A right triangle has an area of 84 m^2 and has integral side lengths. What is the perimeter of this triangle?

3. _____ What is the least positive integer that has a remainder of 0 when divided by 3, a remainder of 1 when divided by 4, and a remainder of 3 when divided by 7?

4. _____ If $u + w = x$ and $w + x = y$ and $x + y = z$, what is the integer value of the sum $u + w + x + y + z$, given that $u = 8$ and $z = 10$?

5. _____ What is the smallest counting number n such that the product $245n$ is a perfect square?

6. _____ (,) A kite is graphed in the coordinate plane as shown. If this kite is rotated 180° clockwise about point $C(0, 1)$, then translated down 4 units and to the left 2 units, what are the coordinates of the final image of point $B(2, 3)$?



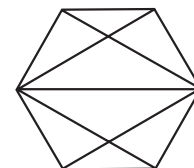
7. _____ %



In 1973, a gallon of gasoline sold for 40 cents. In 2008, a gallon of gasoline sold for \$3.75. By what percent did the cost per gallon of gasoline increase? Express your answer to the nearest tenth.

8. _____ Jason was describing a spinner to his friend. He said the spinner was divided into six equal sections, each containing an integer that was not necessarily distinct from the other integers on the spinner. He told his friend that on the spinner the probability of landing on 2 is $\frac{1}{6}$, the probability of landing on 6 is $\frac{1}{6}$, the probability of landing on a factor of 27 is $\frac{2}{3}$, and the sum of the odd numbers on the spinner is 16. What is the product of the six numbers on the spinner?

9. _____ triangles Using only lines that are already drawn, how many triangles are in the regular hexagon shown?



10. _____ inches What is the length, in inches, of a rectangle that has an area of 18.75 sq in and a length that happens to be $\frac{3}{8}$ of the perimeter? Express your answer as a decimal to the nearest tenth.



Warm-Up 9

1. _____ cm^3 A certain object has a volume of 216 cubic centimeters and a surface area of 240 square centimeters. Another object is a smaller-scale replica of the original object. If the surface area of the smaller object is 60 square centimeters, what is its volume?

2. _____ Five thousand runners will participate in the Mountain Marathon. Each runner will randomly be given a different number from 1 to 5000 to wear during the race. What is the probability that the number of the second-place finisher will be both greater than the number of the third-place finisher and less than the number of the first-place finisher? (Assume no ties.) Express your answer as a common fraction.



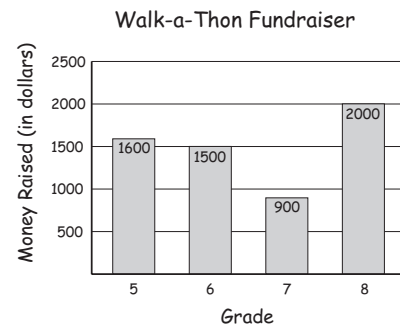
3. _____ marbles Tricia has a bag of marbles. She gave one-third of them to Barbara and one-fourth of the remaining marbles to Sam. If there are now 24 marbles in the bag, how many marbles did Tricia give to Barbara?

4. _____ A cube has a side length of 50 inches. What is the positive difference between the numerical values of the surface area of the cube in square inches and the volume of the cube in cubic inches?

5. _____ units The point $P(1, 4)$ is reflected over the line $y = x$ to point P' . What is the distance between P and P' ? Express your answer in simplest radical form.

6. _____ Think of the set of positive two-digit integers having two different digits that are selected from the digits 2, 3, 4 and 7. In this set, what is the ratio of prime numbers to composite numbers? Express your answer as a common fraction.

7. _____ degrees The students at Valley Middle School held a Walk-a-Thon to raise money for a local charity. The bar graph shows how much money was collected by each grade. If the data had been graphed in a pie chart instead, how many degrees would be in the central angle of the sector representing grade 7?



8. _____ If the seven letters of ALABAMA are to be arranged at random, what is the probability that all four As will be together? Express your answer as a common fraction.

9. _____ units^2 A trapezoid with vertices at $(-1, 2)$, $(1, 2)$, $(3, -2)$ and $(-3, -2)$ is graphed in the coordinate plane. What is the area of the region of the trapezoid where $x \geq 0$?

10. _____ What is the product of the roots of $64x + 16x^2 - 4x^3 - x^4 = 0$?



Warm-Up 10

1. _____ minutes Kevin can mow a lawn in 40 minutes. When Josh helps him, they can finish the lawn in 15 minutes. How long would it take Josh to mow the lawn by himself?



2. _____ In the addition problem shown, A, B, C and D represent distinct digits.

$$\begin{array}{r} ABC \\ +DBBB \\ \hline 2011 \end{array}$$

What is the value of $A + B + C + D$?

3. _____ What is the probability that a positive three-digit integer that has a 3 in the units place is divisible by 3? Express your answer as a common fraction.

4. _____ cu in



A 96-inch piece of wire is cut into pieces, and then the pieces are to be glued together to form the edges of a box. What is the largest possible volume of a box that could have those pieces for edges?

5. _____ quarts A juice drink contains only 10% juice. How many quarts of pure juice must be added to 1 gallon of the juice drink to create a drink that is 60% juice?



6. _____ factors How many factors does 1800 have that are greater than 15?

7. _____ Two cubes, each with faces with 0 through 5, are to be rolled. What is the probability that the sum will be less than 6? Express your answer as a common fraction.

8. _____ The mean, median and range are all the same for a particular set of four numbers. What is the largest number in the set if the smallest number is 17?

9. _____ units² The diagonals of a square intersect at the point (3, 2). The coordinates of one of the vertices of the square are (5, 4). What is the area of this square?

10. _____ In the multiplicative magic square shown, the product of the three numbers in each row, in each column and along each diagonal is 1. What is the value of $r + s$? Express your answer as a common fraction.

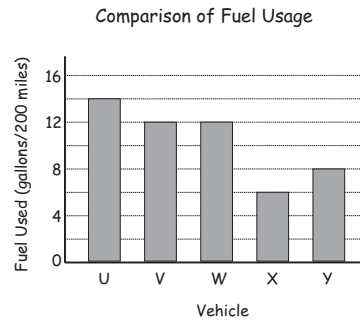
p	q	r
s	1	t
u	4	$\frac{1}{8}$



Workout 5

1. _____ days A painter takes two days to paint a room (all four walls and the ceiling). If he works at the same pace, how many days will it take him to paint a room that is twice as wide, twice as long, and twice as high?

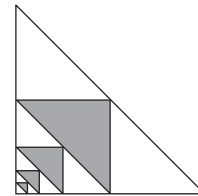
2. _____ mpg The graph shows the amount of fuel used by five different vehicles that each drove 200 miles during a road test. What is the average miles per gallon for the test? Express your answer as a decimal to the nearest tenth.




3. _____ units What is the length of the side of the only equilateral triangle whose perimeter has the same numerical value in units as its area in square units? Express your answer as a decimal to the nearest tenth.

4. \$ _____ Three books and eight CDs cost twice as much as one book and five CDs. If one book and one CD cost \$45, what is the cost of six books?

5. _____ units² If the largest triangle in this figure is a right isosceles triangle with an area of 1 square unit, and if all of the triangles are similar, what is the combined area of the shaded regions? Express your answer as a common fraction.

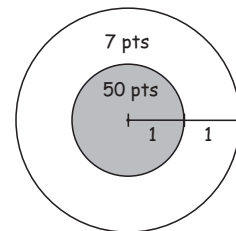


6. _____ What is the sum of the integer values of x , where $x < 8$, that produce an integer value for the expression $\sqrt{2 + 4 + 6 + x}$?

7. _____ %  Assume a diamond's value is directly proportional to the square of its weight in carats. A huge diamond weighing 10 carats is broken into a 4-carat piece and a 6-carat piece. The sum of the values of the two pieces is what percent of the original value?

8. _____ A two-year study was just completed. The project began with 40 participants and ended with 90 participants. After the first year, P participants were in the study. If the percent increase of the number of participants was the same for both years, what is the value of P ?

9. _____ When a dart hits within the center circular region of this circular target, the score is 50, and when a dart hits within the outer ring, the score is 7. If three darts randomly hit the target, what is the probability that the total score is 64? Express your answer as a common fraction.



10. _____ sq units A square with a diagonal of length 7 units is inscribed in a circle. What is the positive difference between the area of the square and the area of the circle? Express your answer as a decimal to the nearest hundredth.

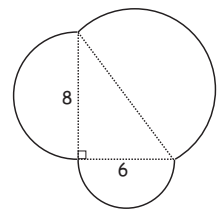



Warm-Up 11



- _____ free throws In basketball, a player can score via 3-point shots, 2-point shots and 1-point free throws. If Shakeel made eight 2-point shots and scored 30 points in all, what is the minimum number of free throws he could have made?
- _____ m² The perimeter of a rectangle is 62 meters. The length is 1 meter more than four times the width. What is the area of the rectangle, in square meters?
- _____ A line segment AB has endpoints at A(5, 9) and B(-7, -15). What is the sum of the coordinates of its midpoint?
- _____ If P is divided by R, the result is $\frac{2}{3}$. If R is divided by S, the result is $\frac{3}{4}$. What is the result if P is divided by S? Express your answer as a common fraction.

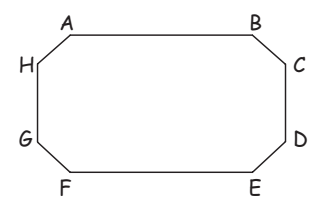
- _____ units² The region shown is composed of a right triangle and three semicircles. What is the area of the entire enclosed region if the triangle has legs of 6 and 8 units? Express your answer in terms of π .

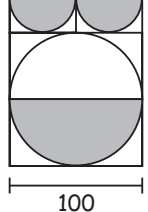


- _____  Julie collects toy frogs. She displays them on a shelf and rearranges them often. If there are 3 green frogs and 2 yellow frogs lined up on her shelf, what is the probability that the frogs on both ends will be green? Express your answer as a common fraction.

- _____ What is the product of all integer values of x for which $|x^2 - 9|$ is a prime number?

- _____ Convex octagon ABCDEFGH has congruent angles A, B, E and F and congruent angles C, D, G and H. If $m\angle A = (2x)^\circ$, $m\angle B = y^\circ$ and $m\angle C = (x + 50)^\circ$, what is the value $x + y$?



- _____ units²  A square of width 100 units contains a circle tangent to all four sides of the square. Two smaller squares, congruent to each other and each containing a circle tangent to all four sides of the square, rest atop the large square, with the three squares together forming a rectangle. What is the difference between the area of the shaded semicircle and the combined area of the two smaller shaded circles?

- _____ ways Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly three elements. Each of the elements of S is distinct. In how many ways can this be done, assuming that the order in which the subsets are chosen is irrelevant?



Warm-Up 12

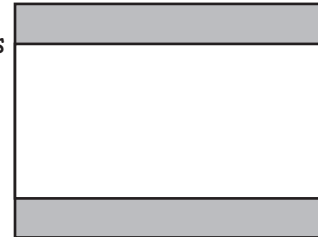
1. _____ The numerator of a fraction is 8 less than the denominator. If the numerator is doubled and the denominator is decreased by 1, the value of the resulting fraction is 1. What is the original fraction? Express your answer as a common fraction.

2. (,) Point A has coordinates $(-2, 5)$ and point B has coordinates $(10, 8)$. Point P is on segment AB, with $AP = 2PB$. What are the coordinates of point P? Express your answer as an ordered pair.

3. _____ What is the value of x if $5^2(5) + 5^2(11) + 5^2(17) + 5^2(78) + 5^2(14) = 5^x$?

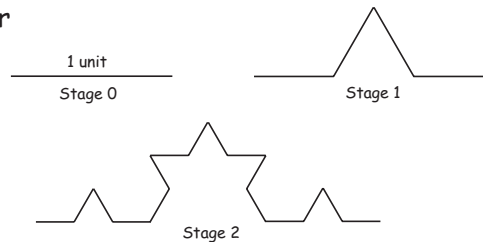
4. _____ The product of two consecutive, positive, odd integers is 783. What is the sum of the two integers?

5. _____ % The aspect ratio of a rectangle is its length divided by its width. The figure shows an image (the white rectangle) with aspect ratio 16:9 displayed on a screen (the largest rectangle) with aspect ratio 4:3. What percent of the area of the screen is occupied by the image?

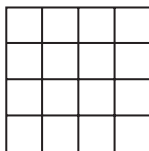


6. _____ The mean of the squares of three non-zero numbers a , b and c is three times the square of their mean. If $b = 2a$, what is the value of $\frac{c}{a}$? Express your answer as a common fraction.

7. _____ $\frac{\text{units}}{\text{longer}}$ When creating a particular pattern, the designer begins with a line segment 1 unit long. Each successive stage is created by using congruent segments that are half as long as those in the previous stage. If you trace the design at Stage 4, how many units longer is it than the length of the design at Stage 1?

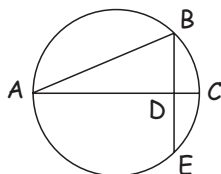


8. _____ Two different unit squares are randomly selected from the 16 unit squares in the 4×4 grid shown. What is the probability that they do not have a vertex in common? Express your answer as a common fraction.



9. _____ The graphs of $y = x^2 - 8x - 35$ and $y = -2x^2 + 16x + 3$ intersect in two points. What is the sum of the x -coordinates of the two points of intersection?

10. _____ $\frac{\text{units}}{\text{units}}$





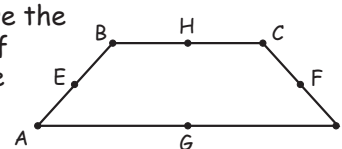
In the figure shown, \overline{AC} is the diameter of the circle. If the measure of arc BC is 60° , $AC \perp BE$, and $BE = 12$ units, what is the length of \overline{AB} ?

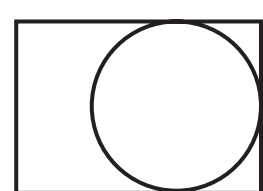


Workout 6

- _____ cents Two apples, five bananas and one carrot cost a total of \$2.05. Three apples, one banana and four carrots cost a total of \$1.89. Three apples, two bananas and three carrots cost a total of \$1.98. What is the total cost, in cents, of one apple, one banana and one carrot?


- _____ points In math class, the mean test score for the girls was 92 points and the mean test score for the boys was 87 points. If 12 girls and 10 boys took the test, what was the mean test score for the whole class? Express your answer as a decimal to the nearest hundredth.
- _____ % Tria paid \$11,500 for a new car after the original price was reduced by a rebate of \$4500. What percentage of the original price did Tria pay for the car? Express your answer to the nearest tenth.
- _____ miles  How many miles can Scotty ride on a bike, going at the rate of 8 miles per hour, if he must walk back to the starting point at a rate of 3 miles per hour (following the same route he traveled on his bike) and he is to be gone a total of 11 hours?
- _____ inches³ In order to send his DVD player back to the manufacturer for repair, Jeremy must pack it securely. He has a packing box with internal dimensions 22" × 18" × 6". The DVD player is a rectangular prism with dimensions 15" × 12" × 2". He has a box with external dimensions 3" × 3" × 8" for the power cord, which also will go inside the packing box. How many cubic inches of packing material must he put in the packing box to fill it completely?
- _____ \$ Stella sells 30 gismos a month at a price of \$20 per gismo. She notices that each time she raises the price of a gismo by \$4, she sells 2 fewer gismos per month. What is the maximum amount of revenue, in dollars, that Stella can make in a month of selling gismos?
- _____ units² Isosceles trapezoid ABCD has bases with lengths 12 and 24. Points E and F are the midpoints of legs AB and CD, respectively. Points G and H are the midpoints of bases AD and BC, respectively. If the height of the trapezoid is 6 units, what is the difference between the areas of trapezoid ABCD and parallelogram HFGE?

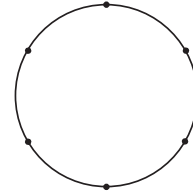

- _____ feet Mr. Kelly is building a 60-foot-long ramp that will be 24 feet high at the highest point. He wants to place a vertical support beam under the ramp with its top 15 feet down the length of the ramp. How long does he need to make the support beam?
- _____ % A circular garden is placed inside a rectangular patio that measures 20 feet by 14 feet. The circular garden is tangent to three sides of the rectangular patio, as shown. What percent of the patio is outside of the garden? Express your answer to the nearest whole number. Use $\frac{22}{7}$ as an approximation for π .


- _____ A bag contains five red marbles, three blue marbles and two green marbles. If six marbles are drawn without replacement from the bag, what is the probability that two marbles of each color are drawn? Express your answer as a common fraction.



Warm-Up 13

1. _____ Six points are equally spaced around a circle. If three of the points are selected at random, what is the probability that they form a scalene triangle? Express your answer as a common fraction.



2. _____ Point P is on the y -axis and is equidistant from the points (1, 3) and (7, 5). What is the y -coordinate of point P?

3. _____ am



A manufacturing plant periodically opens a large valve at the top of a tank to pump water into the tank for use in a manufacturing process. When the valve is opened, water is pumped into the tank at a rate of 75 gallons per minute. Another valve, at the bottom of the tank, sends water out of the tank to an assembly line at a rate of 24 gallons per minute. Both valves are opened simultaneously at 7:30 am and begin filling the empty tank. At what time, to the nearest minute, will the tank be filled to its 600-gallon capacity?

4. _____ A set of 5 positive integers has a mean of 11, a median of 10 and a unique mode of 7. What is the greatest possible range of the set of integers?

5. _____ A jar contains \$5.00 in quarters, \$5.00 in dimes, \$5.00 in nickels and \$5.00 in pennies. If Connie randomly chooses one coin from the jar, what is the probability that the coin chosen is a dime? Express your answer as a common fraction.

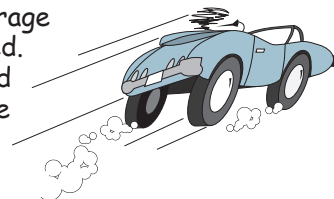
6. _____ Marketing analysts are investigating cylindrical packaging for a new beverage called Star Juice. One member of the design team proposed a can that has a diameter of 6 cm and a height of 20 cm. Another suggested a can that is 17 cm tall and has a diameter of 8 cm. What is the ratio of the smaller volume to the larger volume? Express your answer as a common fraction.

7. _____ values For how many integer values of x is the following inequality true? $-5 \leq 2x - 7 \leq 21$

8. _____ The figure shown is a net of a cube. When folded to form a cube, what is the largest sum of two numbers on opposite faces?



9. _____ miles Yesterday Alice drove one hour longer than Bob at an average speed that was five miles per hour faster than Bob's speed. Clark drove two hours longer than Bob at an average speed that was ten miles per hour faster than Bob's speed. Alice drove 50 miles more than Bob. How many more miles than Bob did Clark drive?



10. _____ units In right triangle ABC, the median to the hypotenuse has length 15 units and the altitude to the hypotenuse has length 12 units. What is the length of the shorter leg of triangle ABC? Express your answer in simplest radical form.

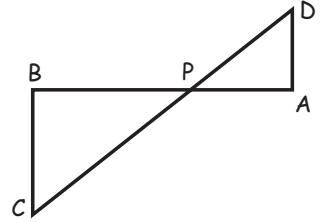


Warm-Up 14

1. _____ In base b , $321 - 123 = 154$. What is the value of b ?

2. _____ What is the value of n if $4(3^{n+2}) + 45(3^n) = 3$?

3. _____ cm In the figure shown, $\overline{AD} = 4$ cm, $\overline{BC} = 8$ cm and $\overline{CD} = 20$ cm. Also, $\overline{AD} \perp \overline{AB}$ and $\overline{BC} \perp \overline{AB}$. What is the length of \overline{AP} ? Express your answer as a common fraction.



4. _____ A rectangle with sides of length 5 units and 6 units that are parallel to the x -axis and y -axis has one vertex at $(1, 3)$. What is the largest possible sum of the coordinates of any of the other vertices of this rectangle?

5. _____ What is the units digit of N in the following equation? $\frac{3 \times 3 \times 3 \times 3 \times 3}{N} = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}$

6. _____ tricycles The cycle shop sells only bicycles and tricycles. All the bicycles have two wheels and all the tricycles have three wheels. If you add up the total wheels on the 67 cycles in the shop, you get 157 wheels. How many tricycles are in the shop?



7. _____ cm² If ABCDEFGH is a regular octagon with sides of length 2 cm, what is the area, in square centimeters, of triangle ABC? Express your answer in simplest radical form.

8. _____ minutes Running at 12 miles per hour burns about 1200 calories per hour. Running at 5 miles per hour burns about 450 calories in an hour. How many minutes longer does one have to run to burn 300 calories at 5 miles per hour than at 12 miles per hour?



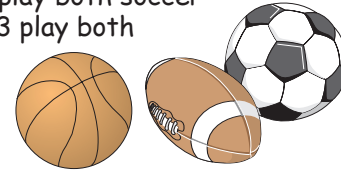
9. _____ students Of the 2000 students in Houston High School, 70% are male. How many male students must be moved to another school so that 60% of the Houston High student body will be males?

10. _____ Suppose that six brothers (George, John, Thomas, Fred, Andrew, Martin) each roll a fair 6-sided die. At least three of them have rolled an even number. What is the probability that John, Fred and Martin have each rolled an even number? Express your answer as a common fraction.



Workout 7

1. _____ students At Marsh Mel Low Middle School 34 students play soccer, 36 students play football and 29 students play basketball. Of these students 15 play both soccer and football, 18 play both basketball and football and 13 play both soccer and basketball. What is the smallest possible number of students that play all three sports?



2. _____ If x and y are two triangular numbers less than 100 that when added, produce a sum that is a square number, what is the largest possible value of x or y ?

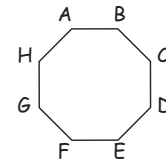
3. _____ gallons Tria's new car can get 50% more miles per gallon of gasoline than her old car could. If her new car gets 24 miles per gallon, how many gallons of gasoline will she save using her new car to travel 480 miles to visit relatives in comparison with driving the old car the same distance? Express your answer to the nearest whole number.

4. _____ hrs Mr. Davidson lives 30 miles from his office. Driving to work on Monday, he averaged 50 mph. However, during his return trip from work, there was construction that slowed traffic. Mr. Davidson averaged only 20 mph going home. How many hours did Mr. Davidson spend traveling to and from work on Monday? Express your answer as a decimal to the nearest tenth.



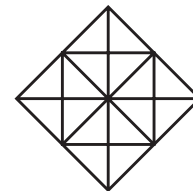
5. _____ Exactly one of the following statements is true. Which one?
 A. Exactly one of these statements is false.
 B. Exactly two of these statements are false.
 C. Exactly three of these statements are false.
 D. Exactly four of these statements are false.
 E. Exactly five of these statements are false.

6. _____ In regular octagon ABCDEFGH, what is the ratio of the area of triangle ADF to the area of triangle AHF? Express your answer as a decimal to the nearest hundredth.



7. \$ _____ If \$1000 is invested at 6% annual interest compounded semiannually, how much interest has been earned at the end of one year?

8. _____ rect How many rectangles are there in the following figure?



9. _____ ways Four children (Alice, Brad, Cathy and Dan) are arranged in a line. If Brad and Cathy cannot be next to each other, in how many ways can the four children be arranged?

10. _____ units² In the coordinate plane, what is the area of the region that is above the x -axis, between $x = -3$ and $x = 3$, and below $y = ||x| - 1| - 2$?



Warm-Up 15

1. _____ packages



Fantasy gum is sold in 3-ounce and 10-ounce packages. One pound is equal to 16 ounces. If 76 packages were sold at B-Rite Mart last month and the total weight sold was less than 16 pounds, what is the greatest number of 10-ounce packages B-Rite could have sold?

2. _____ Three standard, six-sided dice are rolled. The sum of the three numbers on top is less than 6. What is the probability that all three dice show a 1? Express your answer as a common fraction.



3. _____ A line through point $P(4, 3)$ is parallel to the line $x - 2y = 8$. What is the equation of the line through point P ? Express your answer in slope-intercept form where the slope is a common fraction.

4. _____ ways How many different ways can the letters in the name HANK be scrambled so that the newly created "words" start and end with a consonant and are not HANK?

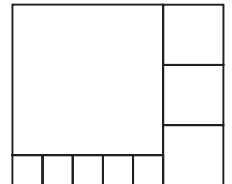
5. _____ sub-committees Eight sophomores are on a student council committee. A party planning subcommittee must be selected from these eight student council committee members. How many different party planning subcommittees are possible if it must have more than 1 but less than 5 members?



6. _____ pairs A regular hexagon and a regular octagon have the same perimeter. If the side lengths of these polygons are whole numbers and the sum of the perimeters is less than 200, how many different pairs of hexagons and octagons can be made?

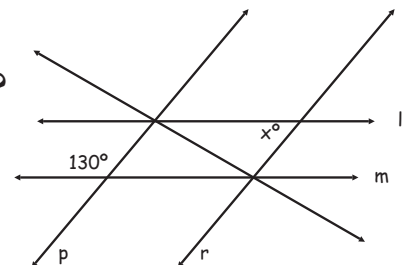
7. _____ What is the integer closest to $\sqrt{150} + \sqrt{75}$?

8. _____ units² A rectangle has been partitioned into nine squares, as shown. If the area of each of the three medium squares is 100 square units, what is the area of the largest square?



9. _____ cm² A circle with a radius of $4\sqrt{3}$ cm is inscribed inside a regular hexagon. What is the area of the hexagon? Express your answer in simplest radical form.

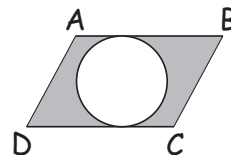
10. _____ In the figure, $l \parallel m$ and $p \parallel r$. What is the value of x ?





Warm-Up 16

1. _____ units² In this figure, side AB of parallelogram ABCD is 10 units long. A circle of radius 3 units is drawn in the interior such that sides AB and CD of the parallelogram are tangent to it. What is the area of the shaded region? Express your answer in terms of π .



2. _____ If $f(x) = 5x$, and $g(x) = f(x) - 3x - 7$, what is the value of $g(8)$?

3. _____ The sum of the digits of a two-digit positive integer is 8. When the digits are reversed, the new integer is 3 more than 4 times the original integer. What is the new integer?

4. _____ degrees The largest angle of a quadrilateral is four times its smallest angle. Another angle is 10 degrees more than twice the smallest. The fourth angle is 60 degrees less than 3 times the smallest. What is the measure of the largest angle?

5. _____ students At Melville Middle School 73 students are in the band, 65 students are in the chorus and 114 students play sports. Thirty-two are both in the band and play sports, 12 play sports and sing in the chorus, but no one is in both the band and the chorus. Five hundred fifty-eight students do not participate in any of these activities. How many students are there at Melville Middle School?



6. _____ sides How many sides does a convex polygon with 20 diagonals have?

7. _____ % On the number line, what percent of the interval $-10 \leq x \leq 10$ satisfies the inequality $x + 2 < \frac{5}{x - 2}$?

8. _____ If $a \Delta b = \frac{a - b}{2b + 6}$, what is the value of $3 \Delta (5 \Delta 3)$? Express your answer as a common fraction.

9. _____ integers How many integers n satisfying $3000 \leq n \leq 4000$ have the product of their digits equal to zero?

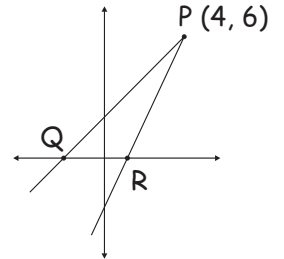
10. _____ Let PQRS be an isosceles trapezoid with bases $PQ = 100$ units and $RS = 26$ units. Suppose $PS = QR = x$ units and a circle with center on base PQ is tangent to both segments PS and QR. If x is the smallest possible value, then what is the value of x^2 ?



Workout 8

- _____ inches Each of Jonathan's bicycle tires revolved 2327 times as he rode three miles. What is the diameter of one of Jonathan's bicycle tires, in inches? Express your answer to the nearest whole number.
- _____ times Six friends A, B, C, D, E and F eat dinner at a local restaurant. A eats there every day. B eats there every other day. C eats there every third day. D eats there every fourth day. E eats there every fifth day. F eats there every sixth day. If all six of them eat together at the restaurant on January 15, how many more times will they eat together in that calendar year?
- _____ liters The kettle in Kerry's kitchen is 80% full of water. After 25% of the water in it has been poured out, there are 1200 ml of water left. How many liters of water does Kerry's kettle hold when it is full?
- _____ inches² A square is inscribed in a circle with radius 6 inches. What is the area inside the circle but outside the square? Express your answer to the nearest whole number.
- _____ % Next week, to celebrate 50 years in business, the grocery store is giving away a bag of free groceries to every 150th customer, starting with the 150th customer. If 2000 customers shop at the store next week, what is the percentage of customers who will receive a free bag of groceries? Express your answer to the nearest hundredth.
- \$ _____ Tom buys two shirts and one pair of socks for \$15. Sam buys one shirt and two pairs of socks for \$12. How much will Jim pay for 15 shirts and 10 pairs of socks?
- _____ feet In isosceles triangle ABC, $AB = AC = 13$ ft, and $BC = 10$ ft. Square RSTU is inscribed in the triangle, with R and S on \overline{BC} , with T on \overline{AC} and with U on \overline{AB} . What is the length of a side of the square? Express your answer as a common fraction.

- _____ units² A line with slope equal to 1 and a line with slope equal to 2 intersect at the point $P(4, 6)$, as shown. Points Q and R are both on the x-axis. What is the area of triangle PQR?



- _____ When Andy, Becky, Chad, Dave, Eleanor and Fred line up in a row, what is the probability that Andy and Becky are not standing next to each other? Express your answer as a common fraction.



- _____ units² In convex pentagon JKLMN, $KL = LM = MN = 4$ units and $JK = JN = 8$ units. If $m\angle J = 60^\circ$ and $m\angle L = m\angle M$, what is the area of the pentagon, expressed in simplest radical form?



Warm-Up 17



1. _____ days For every day his homework is late, Sam's grade is reduced by 10%. For example, if Sam's homework is 2 days late, his score will be multiplied by 0.8. Sam will get 4 more problems correct for each extra day he works on his homework past the due date. If Sam has 14 out of 30 problems correct the day his assignment is due, how many days late should he turn it in to get the maximum grade?

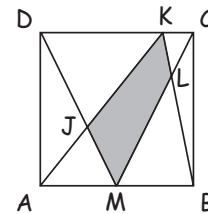
2. _____ ways Eight identical marbles are to be placed in five boxes numbered 1 through 5 so that each box contains at least one marble. In how many ways can this be done?

3. _____ Two natural numbers have a greatest common factor of 3 and a least common multiple of 216. If the difference between the two numbers is 3, what is the sum of the two numbers?

4. _____ The table lists points (x, y) that satisfy the linear relation $y = mx + b$. What is the product of m and b ? Express your answer as a common fraction.

x	1	5	9	13
y	2	5	8	11

5. _____ units² Square $ABCD$ has sides of length 4 units. M is the midpoint of \overline{AB} , and K is on \overline{CD} so that $DK:KC = 3:1$. \overline{DM} and \overline{AK} intersect at J . \overline{MC} and \overline{BK} intersect at L . What is the area of quadrilateral $JKLM$? Express your answer as a common fraction.



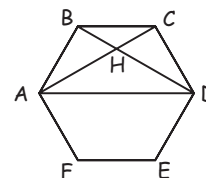
6. _____ Given that $n > 1$, what is the smallest positive integer n whose divisors have a product of n^8 ?

7. _____ units² In triangle RST , J is on line segment RT , with $RJ:JT = 2:1$. Also, K is on line segment ST , with $TK:KS = 2:1$. Line segments SJ and RK intersect at point P . If the area of triangle SPK is 7 units², what is the area of triangle RPS ?

8. _____ minutes At 3:00, the hour hand and minute hand of a 12-hour clock are perpendicular. What is the least number of minutes that must elapse for this to be true again? Express your answer as a mixed number.

9. _____ balls One hundred red balls are lined up in a row. Starting from the left end, every fourth ball is replaced with a green ball. Then, starting from the right end, every fifth ball is replaced with a white ball. Finally, starting from the left end, every sixth ball is replaced with a yellow ball. How many red balls remain in the row?

10. _____ If $ABCDEF$ is a regular hexagon, what is the ratio of the area of triangle AHD to the area of hexagon $ABCDEF$? Express your answer as a common fraction.

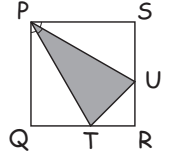




Warm-Up 18

- _____ What is the product of x and y if $x^2 + y^2 = 36 - 2xy$ and $x^2 - y^2 = 12$?
- _____ For each positive two-digit integer, John adds the two digits. For example, 34 gives $3 + 4 = 7$. What is the sum of all of his results?

- _____ units^2 The figure shows $\triangle P TU$ inscribed in square $PQRS$. If $PQ = 12$ units and segments PT and PU trisect $\angle QPS$, what is the area of $\triangle P TU$?



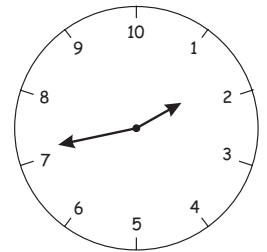
- _____ The mean, median and range are all 6 for a collection of five positive integers. What is the sum of all possible distinct values that could be the greatest number in all the possible collections?
- _____ What is the sum of the reciprocals of the roots of $2x^2 + 3x - 8 = 0$? Express your answer as a common fraction.

- _____ This Sudoku-like figure is a 4×4 grid to be filled so that each of the digits 1, 2, 3 and 4 appears in each row and in each column. The 4×4 grid is divided into four 2×2 squares. Each of these 2×2 squares also is to contain each of the digits 1, 2, 3 and 4. What digit replaces P?

1		3	
	2		
	P		
			4

- _____ A small square is inscribed in a circle that has another, larger square circumscribed about it. What is the ratio of the area of the small square to the area of the large square? Express your answer as a common fraction.
- _____ units^2 Three rectangles have the same area. Their dimensions are m by n , $(m + 2)$ by $(n - 2)$ and $(m - 2)$ by $(n + 10)$. What is the area of each rectangle?

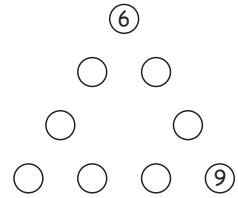
- _____ minutes Ben came up with a new idea for a clock and telling time. He wants to measure a day as two 10-"hour" cycles rather than two 12-hour cycles. However, Ben does not want to change the duration of a minute, so a day would still have the same number of minutes. How many minutes are there in a "Ben-clock hour"?



- _____ cubes Two red, two yellow and two green faces, all unit squares, are available for building a cube. How many distinct cubes can be built?



Workout 9



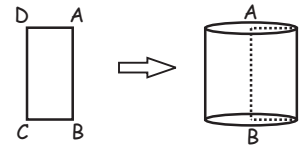
- _____ Each of the integers 1 through 9 is to be placed in one of the nine given circles so that the sum, S , of the four numbers along each side of the triangle is the same. Two of the numbers have been entered. What is the value of S ?
- _____ units In trapezoid $ABCD$, bases AB and CD have lengths 10 and 6, respectively. Line segment PQ is drawn parallel to the bases, with P on \overline{AD} and Q on \overline{BC} so that the area of $ABQP$ is three-fourths that of $ABCD$. What is the length of PQ ? Express your answer in simplest radical form.
- _____ ways The number 20 can be expressed as a sum of three natural numbers in many ways. Three distinct examples are $1 + 12 + 7$, $3 + 14 + 3$ and $14 + 3 + 3$. Including the examples shown, in how many distinct ways can 20 be expressed as a sum of three natural numbers?
- _____ mm^3 Base BCD of right tetrahedron $ABCD$ is an equilateral triangle with sides of length 6 mm. Each of the lateral sides of the tetrahedron is an isosceles right triangle. What is the volume of the tetrahedron? Express your answer in simplest radical form.
- _____ numbers Of the first 2011 natural numbers, how many have exactly three digits of 1 when written in base 2 form?
- _____ cm^2 Each side of equilateral triangle ABC has length 60 cm. Point U is on \overline{AB} , with $AU:UB = 1:2$; point V is on \overline{BC} with $BV:VC = 1:3$; and point W is on \overline{AC} , with $CW:WA = 1:4$. What is the area of triangle UVW ? Express your answer in simplest radical form.
- _____ miles On Sunday, John drove from his house to his uncle's house for a visit. If his average speed had been 10 miles per hour slower, the trip would have taken 2 hours longer. If his average speed had been 20 miles per hour faster, the trip would have taken 2 hours less. How many miles is it from John's house to his uncle's house?
- _____ If a positive two-digit integer is c times the sum of its digits, the number formed by interchanging the digits is the sum of the digits multiplied by what expression that involves c ?
- _____ ordered pairs How many ordered pairs of integers (x, y) satisfy $x^2 + y^2 = 65^2$?
- _____ There are several natural numbers, n , such that $B = n^2 + 2008n$ and B has a units digit of 4. What is the largest possible digit that can be in the tens place of B ?



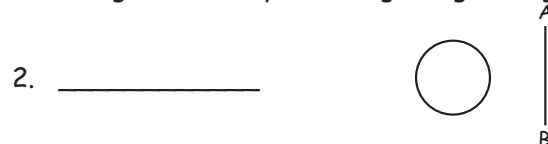
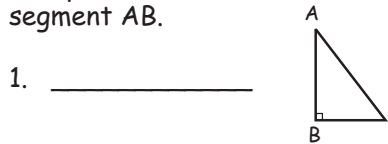


Solids Stretch

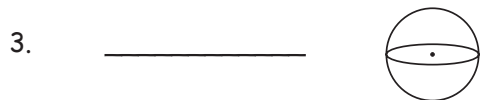
If you revolve rectangle ABCD about side AB, you will generate a cylinder. Line AB is the axis of revolution for this solid of revolution.



For questions 1 and 2, describe the solid of revolution generated by revolving the given figure 360° about segment AB.

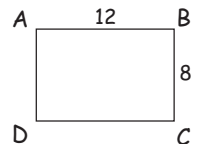


For questions 3 and 4, what plane figure will generate the given solid when revolved 360° about an axis of revolution? Draw or describe the figure and indicate the axis of revolution.



5. If you revolve this rectangle about side AB or side BC, you form a cylinder.

- a. Revolving around which of these two segments produces a cylinder with a larger volume? _____
- b. Revolving around which of these two segments produces a cylinder with a larger total surface area? _____

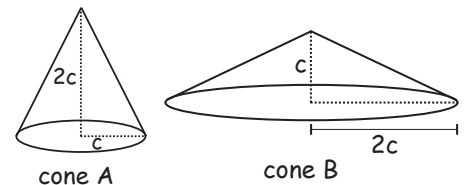



6. _____ Consider this shaded rectangle with $b > a$. Let V_a denote the volume of the cylinder that is created when the shaded rectangle is revolved around side a , and let V_b denote the volume of the cylinder that is created when the shaded rectangle is revolved around side b . What is the value of the ratio $\frac{V_b}{V_a}$? Express your answer as a common fraction in terms of a and b .



7. _____ Let S_a denote the total surface area of the cylinder that is created when the shaded rectangle used in question 6 is revolved around side a , and let S_b denote the total surface area of the cylinder that is created when the shaded rectangle is revolved around side b . What is the value of the ratio $\frac{S_b}{S_a}$? Express your answer as a common fraction in terms of a and b .

8. _____ These cones can be formed by revolving a right triangle about its two legs. What is the ratio of the volume of cone A to the volume of cone B? Express your answer as a common fraction.



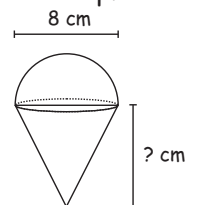
9.  The side lengths in the figure are given. What is the volume of the solid that results from revolving the triangle 360° around . . .

- a) side AC? Express your answer in terms of π . _____ units³
- b) side BC? Express your answer in terms of π . _____ units³
- c) side AB? Express your answer as a common fraction in terms of π . _____ units³

10. An ice cream cone is packed full of ice cream, and a hemisphere of ice cream is placed on top.

a. What plane figure will generate this solid when revolved 360° about an axis of revolution? Draw or describe the figure and indicate the axis of revolution. _____

b. If the volume of ice cream inside the cone is the same as the volume of ice cream outside the cone, how many centimeters is the height of the cone? _____ cm





Sum and Product SUPER Stretch

- _____ What is the sum of the solutions of $6x^2 + 5x - 4 = 0$? Express your answer as a common fraction.
- _____ A quadratic equation of the form $x^2 + kx + m = 0$ has solutions $x = 3 + 2\sqrt{2}$ and $x = 3 - 2\sqrt{2}$. What is the value of $k + m$?
- _____ What is the sum of the reciprocals of the solutions of $4x^2 - 13x + 3 = 0$? Express your answer as a common fraction.
- _____ If r and s are the solutions of $2x^2 + 9x + 3 = 0$, what is the value of $r^2 + s^2$? Express your answer as a common fraction.
- _____ If r and s are the solutions of $x^2 + 6x - 2 = 0$, what is the value of $r^3 + s^3$?
- (_____ , _____) The solutions of $x^2 + bx + c = 0$ are each 5 more than the solutions of $x^2 + 7x + 3 = 0$. What are the values of b and c ? Express your answer as an ordered pair (b, c) .
- (_____ , _____ , _____) A cubic equation of the form $x^3 + bx^2 + cx + d = 0$ has solutions $x = 3$, $x = 4$ and $x = 5$. What are the values of b , c and d ? Express your answer as an ordered triple (b, c, d) .
- _____ What is the sum of the reciprocals of the solutions of $x^3 - 3x^2 - 13x + 15 = 0$? Express your answer as a common fraction.
- _____ What is the sum of the squares of the solutions of $x^3 - 15x^2 + 66x - 80 = 0$?
- _____ The solutions of $x^3 - 63x^2 + cx - 1728 = 0$ form a geometric sequence. What is the value of c ?