Special Triangle Relationships

30-60-90 Triangles

A 30-60-90 triangle is a right triangle whose internal angles are 30, 60 and 90 degrees. The three sides of a 30-60-90 triangle have the following characteristics:

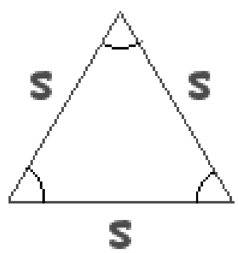
- All three sides have different lengths
- The shorter leg, b, is half the length of the hypotenuse, c. That is,

  $b = c/2$

- The longer leg's length, a, is the shorter leg times $\sqrt{3}$. That is,

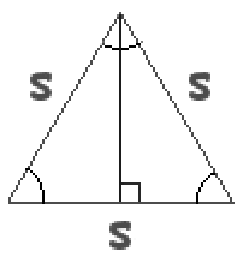
  $a = b\sqrt{3}$

To demonstrate why those points are true we begin with an equilateral triangle.



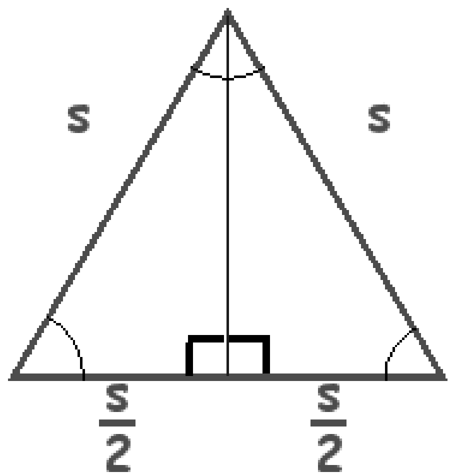
Because the three sides are the same size, it's called an **equi**lateral triangle. Since the three sides are the same, the internal angles are all 60 degrees.

If we drop a perpendicular line from the top of the triangle, we get

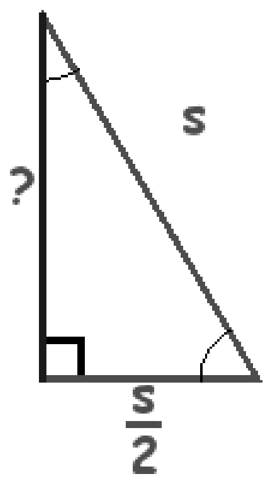


The little box at the bottom next to the new line indicates that the new line is perpendicular to the red line. The word "perpendicular" means the lines form a 90 degree angle. The box is shorthand for a 90 degree angle.

The perpendicular line divides the triangle in half into two identical triangles so the bottom length is split in half as shown below.

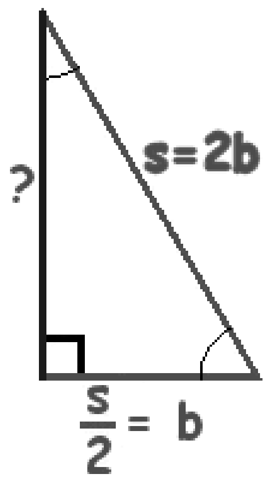


Focusing on the triangle on the right,



we can say that the bottom right angle is 60 degrees and the bottom left angle is 90 degrees. Since the three angles of a triangle add to 180 degrees, the top angle must be 30 degrees. The only remaining question is how tall is the triangle?

If we relabel the short leg $\frac{s}{2}$ = b then the hypotenuse is **2b** because the short leg is half the hypotenuse.



Since the triangle is a right triangle, we can use the Pythagorean formula to find the height,

$$a^2 + b^2 = c^2$$

Substituting for c, we get

$$a^2 + b^2 = (2b)^2$$

where a^2 is the height squared and b^2 is the bottom leg squared.

Expanding the right side, we get

$$a^2 + b^2 = (2b)(2b)$$

Rearranging:

$$a^2 + b^2 = (2)(2)(b)(b)$$

or

$$a^2 + b^2 = 4b^2$$

Subtracting b^2 from both sides we get

$$a^2 = 3b^2$$

Since we want a, not a^2, we take the square root of both sides

$$\sqrt{a^2} = \sqrt{3b^2}$$

We can rewrite that as

$$\sqrt{a^2} = (\sqrt{3})(\sqrt{b^2})$$

We can simplify,

$$a = \sqrt{3}\ b$$

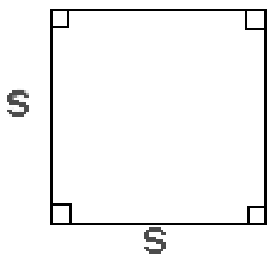
Rearranging,

$$a = b\sqrt{3}$$

which says that the height of this 30-60-90 triangle is the short side times the square root of 3.

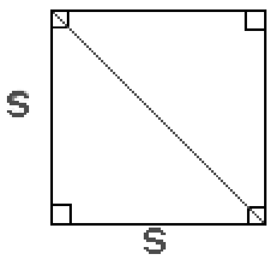
45-45-90 Triangles

The three sides of a 45-45-90 triangle are derived by beginning with a square as shown:

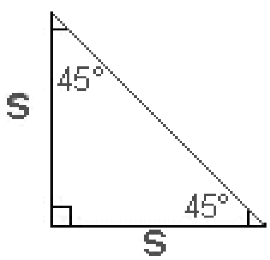


The sides of the squares are identical and are marked as being "s" units long.

We divide the square in half by drawing a diagonal and we get:



The square is divided into two triangles. We focus on the bottom triangle.



The three angles of the triangle are 45, 45 and 90 degrees. Because it is a right triangle, we can use the Pythagorean formula to find the length of the hypotenuse:

$a^2 + b^2 = c^2$

Substituting for the legs we get

$s^2 + s^2 = c^2$

Combining like terms:

$2\mathbf{S}^2 = c^2$

We want the length of the hypotenuse, not its square so we take the square root of both sides and get:

$\sqrt{2S^2}$style="text-indent: 30pt; line-height: 0pt"= style="text-indent: 30pt; line-height: 0pt"$\sqrt{c^2}$

The squares come out from under the radical leaving

$S\sqrt{2} = c$

That means that the hypotenuse of any 45-45-90 triangle is the length of one of the legs times the square root of two. The diagrams also show that both legs of the triangle are identical so if you know one, you know the other leg's length.

Some examples: