
2010–2011

MATHCOUNTS[®]

School Handbook:

Volume I

Contains more than 100 creative math problems
that meet NCTM standards for grades 6-8.

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FORMS OF ANSWERS

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A “common fraction” is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term “common fraction” is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form $\pm N \frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$
Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}, 1\frac{1}{2}$
Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π . *Answer:* $\frac{1+2\pi}{8}$
Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}, \frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}, \frac{3}{\pi}, \frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}, \frac{1}{3}, 3.5, 2:1$

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: Evaluate $\sqrt{15} \times \sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$) $a.bc$, where a is an integer and b and c are digits.

The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the “rounding” a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3
Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.

Thus, 25.0 will not be accepted for 25 nor vice versa.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

abscissa	degree measure	interior angle of a polygon
absolute value	denominator	intersection
acute angle	diagonal of a polygon	inverse variation
additive inverse (opposite)	diagonal of a polyhedron	irrational number
adjacent angles	diameter	isosceles
algorithm	difference	lateral surface area
alternate interior angles	digit	lateral edge
alternate exterior angles	digit-sum	lattice point(s)
altitude (height)	direct variation	LCM
area	dividend	linear equation
arithmetic mean	divisible	mean
arithmetic sequence	divisor	median of a set of data
base 10	edge	median of a triangle
binary	endpoint	midpoint
bisect	equation	mixed number
box-and-whisker plot	equiangular	mode(s) of a set of data
center	equidistant	multiple
chord	equilateral	multiplicative inverse (reciprocal)
circle	evaluate	natural number
circumference	expected value	numerator
circumscribe	exponent	obtuse angle
coefficient	expression	octagon
collinear	exterior angle of a polygon	octahedron
combination	factor	odds (probability)
common denominator	factorial	opposite of a number (additive inverse)
common divisor	Fibonacci sequence	ordered pair
common factor	finite	ordinate
common fraction	formula	origin
common multiple	frequency distribution	palindrome
complementary angles	frustum	parallel
composite number	function	parallelogram
compound interest	GCF	Pascal's triangle
concentric	geometric mean	pentagon
cone	geometric sequence	percent increase/decrease
congruent	height (altitude)	perimeter
convex	hemisphere	permutation
coordinate plane/system	hexagon	perpendicular
coordinates of a point	hypotenuse	planar
corresponding angles	image of a point (under a transformation)	polygon
counting numbers	improper fraction	polyhedron
counting principle	inequality	prime factorization
cube	infinite series	prime number
cylinder	inscribe	principal square root
data	integer	
decimal		

prism	remainder	system of equations/ inequalities
probability	repeating decimal	tangent figures
product	revolution	tangent line
proper divisor	rhombus	term
proper factor	right angle	terminating decimal
proper fraction	right circular cone	tetrahedron
proportion	right circular cylinder	total surface area
pyramid	right polyhedron	transformation
Pythagorean Triple	right triangle	translation
quadrant	rotation	trapezoid
quadrilateral	scalene triangle	triangle
quotient	scientific notation	triangular numbers
radius	segment of a line	trisect
random	semicircle	union
range of a data set	sequence	unit fraction
rate	set	variable
ratio	similar figures	vertex
rational number	simple interest	vertical angles
ray	slope	volume
real number	slope-intercept form	whole number
reciprocal (multiplicative inverse)	solution set	x -axis
rectangle	sphere	x -coordinate
reflection	square	x -intercept
regular polygon	square root	y -axis
relatively prime	stem-and-leaf plot	y -coordinate
	sum	y -intercept
	supplementary angles	

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle $C = 2 \times \pi \times r = \pi \times d$

AREA

Square $A = s^2$

Rectangle $A = l \times w = b \times h$

Parallelogram $A = b \times h$

Trapezoid $A = \frac{1}{2}(b_1 + b_2) \times h$

Circle $A = \pi \times r^2$

Triangle $A = \frac{1}{2} \times b \times h$

Triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$

Equilateral triangle $A = \frac{s^2 \sqrt{3}}{4}$

Rhombus $A = \frac{1}{2} \times d_1 \times d_2$

SURFACE AREA & VOLUME

Sphere $SA = 4 \times \pi \times r^2$

Sphere $V = \frac{4}{3} \times \pi \times r^3$

Rectangular prism $V = l \times w \times h$

Circular cylinder $V = \pi \times r^2 \times h$

Circular cone $V = \frac{1}{3} \times \pi \times r^2 \times h$

Pyramid $V = \frac{1}{3} \times B \times h$

Pythagorean Theorem $c^2 = a^2 + b^2$

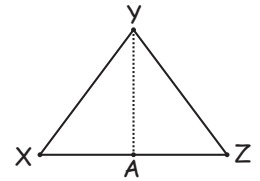
Counting/
Combinations ${}_n C_r = \frac{n!}{(r!)(n-r)!}$

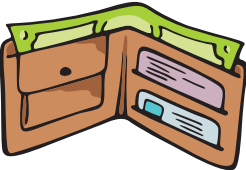


Warm-Up 1

1. \$ _____ If a total of twenty million dollars is to be divided evenly among four million people, how much money, in dollars, will each person receive?

2. _____ sq cm Triangle XYZ has side $XZ = 6$ cm. Segment YA is perpendicular to XZ . If AY is 4 cm, what is the area of triangle XYZ?



3. \$ _____  Wilhelmina went to the store to buy a few groceries. When she paid for the groceries with a \$20 bill, she correctly received \$4.63 back in change. How much did the groceries cost?

4. _____ multiples How many multiples of 8 are between 100 and 175?

5. _____ A three-digit integer is to be randomly created using each of the digits 2, 3 and 6 once. What is the probability that the number created is even? Express your answer as a common fraction.

6. _____ In the following arithmetic sequence, what is the value of m ?

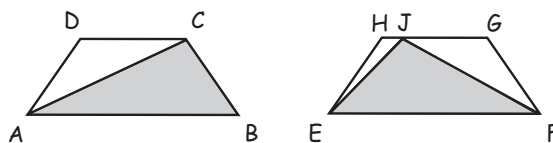
$-2, 4, m, 16, \dots$

7. _____ A particular fraction is equivalent to $\frac{2}{3}$. The sum of its numerator and denominator is 105. What is the numerator of the fraction?

8. _____ What is the least natural number that has four distinct prime factors?

9. _____ areas The perimeter of a particular rectangle is 24 inches. If its length and width are positive integers, how many distinct areas could the rectangle have?

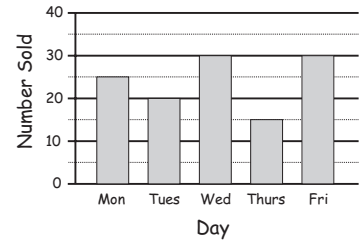
10. _____ sq units Trapezoid ABCD and trapezoid EFGH are congruent. What is the difference between the area of triangle ABC and the area of triangle EFJ?





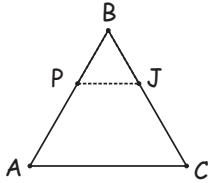
Warm-Up 2

Cartons of Milk Sold per Day



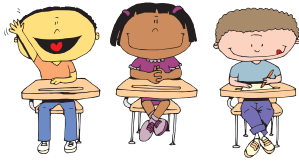
1. _____ cartons The bar graph shows the number of cartons of milk sold per day last week at Jones Junior High. What is the positive difference between the mean and median number of cartons of milk sold per day over the five-day period?

2. _____ inches



An equilateral triangle PBJ that measures 2 inches on each side is cut from a larger equilateral triangle ABC that measures 5 inches on each side. What is the perimeter of trapezoid PJCA?

3. _____ boys



Currently, $\frac{1}{4}$ of the members of a local club are boys, and there are 80 members. If no one withdraws from the club, what is the minimum number of boys that would need to join to make the club $\frac{1}{3}$ boys?

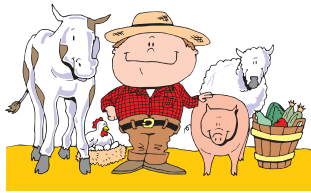
4. _____ outfits If William has 3 pairs of pants and 4 shirts, and an outfit consists of 1 pair of pants and 1 shirt, how many distinct outfits can William create?

5. _____ days Jessica reads 30 pages of her book on the first day. The next day, she reads another 36 pages. On the third day, she reads another 42 pages. If she continues to increase the number of pages she reads each day by 6, how many days will it take her to read a book that has 270 pages?

6. _____ A basket of fruit contains 4 oranges, 5 apples and 6 bananas. If you choose a piece of fruit at random from the basket, what is the probability that it will be a banana? Express your answer as a common fraction.

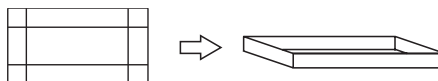
7. _____ What number is halfway between $\frac{5}{8}$ and $\frac{11}{16}$? Express your answer as a common fraction.

8. _____ pounds



Farmer Fred read that his crop needs a fertilizer that is 8% nitrate for optimal yield. He needs to apply 4 pounds of nitrate per acre. If his field has 188 acres, how many pounds of fertilizer will Fred use?


9. _____ sq in Four squares are cut from the corners of a rectangular sheet of cardboard. It is then folded as shown to make a box that is 15 inches long, 8 inches wide and 2 inches tall. What was the area of the original piece of cardboard?



10. _____ degrees Two consecutive angles of a regular octagon are bisected. What is the degree measure of each of the acute angles formed by the intersection of the two angle bisectors?



Workout 1

1. \$ _____ Sweaters cost \$49 at a local department store, and the catalog department of the same store sells them for \$44 online. The local department store charges 7% tax, and the catalog department charges \$3.50 per sweater for shipping (but does not charge tax). How much would a person save by buying 6 sweaters through the catalog department rather than at the department store?
2. _____ mph If Jordan passed mile marker 138 at 5:08 pm and then passed mile marker 216 at 6:20 pm, what was her average speed in miles per hour?
3. _____ pounds A group of 5 women and 8 men weigh a total of 1921 pounds. If the women of the group have an average weight of 109 pounds, what is the average weight of the men in the group?
4. _____ womps On planet Wobble, 1 womp is equal in value to 3 wamps, and 2 wamps are equal in value to 5 wemps. How many womps are equal in value to 15 wemps?
5. _____ cubes How many positive three-digit perfect cubes are even?
6. _____ questions  Billy was playing a trivia game. According to the rules, Billy would receive 25 points for each question he answered correctly but would lose 50 points for each question he answered incorrectly. At the end of the game, Billy had a total of 450 points. He had 5 times as many questions correct as incorrect. If Billy answered every question, how many questions were asked in the game?
7. _____ What is the value of the following expression, expressed as a repeating decimal?
$$\left(\frac{1}{9} + \frac{1}{7}\right) - \frac{2}{9}$$
8. _____ Gary chooses a two-digit positive integer, adds it to 200, and squares the result. What is the largest number Gary can get?
9. \$ _____ The sale price of Kara's dress was \$43.20 after a 40% discount. What was the original price of the dress?
10. _____ degrees The supplement of an angle is 5.5 times the complement of the angle. What is the measure of the angle?





Warm-Up 3

1. _____ Thirty percent of twice a number is 30. What was the original number?

2. _____ inches A bouncy ball bounces 16 feet high on its first bounce and exactly half as high on the next bounce. For each later bounce, the ball bounces half as high as on the previous bounce. How many inches high does the ball bounce on its seventh bounce?



3. \$ _____ Tamika's dad bought a Mustang convertible for \$12,000 to celebrate his first job in 1978, which paid him \$15,000 a year. If Tamika is to use the same fraction of her salary as her dad to buy a Mustang convertible, which now sells for \$32,000, what will her first job have to pay?

4. _____ A rectangular prism has dimensions 4 by 6 by x . If the total surface area of the prism is 248 square units, what is the value of x ?

5. _____ ways How many ways can the letters in FACTOR be rearranged so that the first and last letters are vowels?

6. _____ If $a \# b = (a + b)^2$, what is the value of $3 \# 1$?

7. _____ The triangular array of positive integers shown continues indefinitely, with each row containing one entry more than the row above it. What is the sum of the two integers directly above 100?

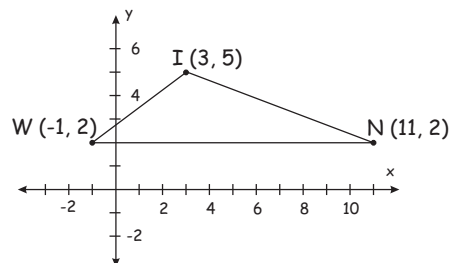
4					
7	10				
13	16	19			
22	25	28	31		
34	37	40	43	46	

8. _____ points In a class of 20 students, the average (arithmetic mean) score on a test is 84 points. If 6 students each scored 100 points and 4 students each scored 50 points, what is the average of the scores of the remaining students?



9. _____ If $\frac{a-2b}{3a-4b} = 5$, what is the ratio of a to b ? Express your answer as a common fraction.

10. (_____ , _____) If triangle WIN is reflected over the x -axis to create triangle W'IN', what will be the coordinates of point I'? Express your answer as an ordered pair.


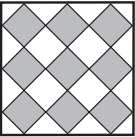

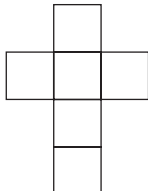




Warm-Up 4

Potato Chip Prices

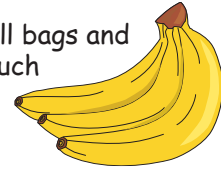
15	9	Key 15 9 means \$1.59
16	9	
17	5 9	
18	8 9	
19	9 9 9 9 9	
20	9	
21	5 5 8 9	
22	5 9 9	
23	9	
24		
25	9	

- bags A survey of the cost of one-pound bags of potato chips at five local supermarkets is shown in the stem-and-leaf plot. How many bags of chips cost less than the median price of a bag of chips?
- $\frac{\text{more}}{\text{rap songs}}$ Vivian has 45 songs on her MP3-player, 15 of which are alternative rock and the rest of which are rap. Malik has 60 songs on his MP3-player in the same ratio as Vivian's songs. How many more rap songs does Malik have on his MP3-player than Vivian has on hers?
- pckgs Lashae will plant pansies and violets around the edge of her garden. She must use the same number of each plant, and she must use all of the flowers she buys. If pansies are sold in packages of 6 and violets are sold in packages of 10, what is the least number of packages of violets she could buy, given that she buys at least 1 package? 
- $\begin{array}{c} 1 \\ 1 \ 1 \\ 1 \ 2 \ 1 \\ 1 \ 3 \ 3 \ 1 \end{array}$ The first four rows of Pascal's triangle are shown to the left. If the first two numbers in a particular row of Pascal's triangle are 1 and 10, what is the fourth number of the next row?
- In the figure, the gray small squares and the white small squares are all the same size, and the outer corners of the outer gray squares lay on the edge of the largest square. What fraction of the largest square is shaded? Express your answer as a common fraction. 
- %  Last July there were 56 dogs for sale in the newspaper ads, and this July there were 84 dogs for sale. What is the percent increase in the number of dogs for sale?
- What is the value of the expression $3x + 4y^3$ if $x = 3$ and $y = -2$?
- units Points A, B and C are collinear, with B between A and C. If the distance from point A to point B is 8 more than twice the distance from point B to point C, and the length of segment AC is 5 times the length of segment BC, how long is segment AC?
- The sum of two numbers is 3 times their positive difference. What is the ratio of the smaller number to the larger number? Express your answer as a common fraction.
- units³ The net shown is composed of 6 congruent squares and has a perimeter of 56 units. When the net is folded into a cube, what is the volume of the cube? 



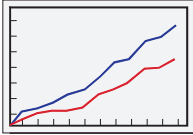
Workout 2

1. \$ _____ Sara has 30 pounds of dried bananas that she wants to divide into small bags and sell. If each small bag holds $\frac{5}{8}$ of a pound and will sell for \$1.58, how much will Sara bring in from sales if she sells all the bags?



2. _____ integers How many five-digit integers are perfect squares?

3. \$ _____ Four friends (Amanda, Barbara, Charles and Dylan) invested \$3000, \$5000, \$7000 and \$9000, respectively, in a certain start-up company. Profits will be divided in the same ratio as the investment. If the business made a profit of \$72,000 this year, how much of the profit will the largest investor receive?

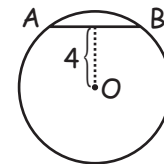


4. _____ points The mean score on Mr. Pascal's first semester exam for his 28 students is 73 points. If he removes the lowest score of 19 points, what is the adjusted class average (arithmetic mean)?

5. _____ The sum of four consecutive even integers is 596. What is the product of the least and greatest of these integers?

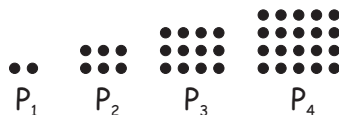
6. _____
- | | | | | |
|--|----|----|----|---|
| | | N | | |
| | | 65 | | |
| | 34 | | | |
| | | | 15 | |
| | | 9 | | 7 |
- In this Number Wall, you add the numbers next to each other and write the sum in the block directly above the two numbers. What number will be in the block labeled N?

7. _____ sq units Chord AB of circle O is 6 units long and 4 units from center O. What is the area of circle O? Express your answer in terms of π .



8. _____ Three fair six-sided number cubes, each with faces labeled 0, 2, 4, 6, 8, 10, are rolled. What is the probability that the sum of the numbers rolled is greater than 20? Express your answer as a common fraction.

9. _____ In the patterns of dots with P_1 , P_2 , P_3 and P_4 , each successive pattern of dots has one more row and one more column. What is the ratio of the number of dots in P_{50} to the number of dots in P_{25} ? Express your answer as a common fraction.



10. _____ people The population of Circletown has been growing at an annual rate of 5% for the last 5 years. The population is now 3105 people. What was the population of Circletown 5 years ago? Express your answer to the nearest whole number.

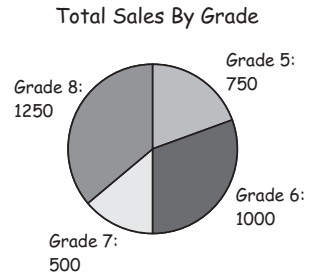




Warm-Up 5

1. _____ If Minh picks a letter of the alphabet at random, what is the probability it occurs in the name "Hattiesburg, Mississippi"? Express your answer as a common fraction.

2. _____% The graph shows the amount of sales made by each grade at Memorial Middle School in the annual fundraiser. What percent of the total sales was made by the Grade 7 class? Express your answer to the nearest tenth.



3. _____ What is the integer value of $0 - 5 + 10 - 15 + 20 - 25 + 30 - \dots + 240$?

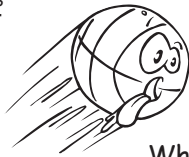
4. _____ In this additive magic square the sum of the three numbers in each row, in each column and along each diagonal is the same. What is the value of z ?

		z
11	6	
		10

5. _____ sq units What is the area of the quadrilateral with vertices at $(1, 1)$, $(4, 1)$, $(7, 5)$ and $(4, 5)$?

6. _____ If $a \# b = a^b - b^a$, what is the value of $3 \# 5$?

7. _____%



Coach Kennedy had the starters of his basketball team practice free throws. Each player had to shoot 20 free throws. On Wednesday, 4 of the 5 starters did the free throw drill and averaged making 15 out of 20 attempted baskets. When the fifth starter did his drill, the team average dropped to 14 made out of 20 attempted. What percent of his free throws did the fifth starter make?

8. _____ ft per sec William is traveling at a rate of 75 miles per hour. How fast is William traveling in feet per second?

9. _____ pounds To create a more affordable line of her gourmet chocolates, Kindra mixes 10 pounds of chocolate worth \$1.50 per pound with another type of chocolate, worth \$9.00 per pound. She wishes to make a mixture of chocolate worth \$4.00 per pound. How many pounds of the \$9.00 chocolate must she add to create her new product?



10. _____ inches A solid is formed by placing a square pyramid with a base of 3 inches by 3 inches on top of a cube with edges of 3 inches. The volume of this combined solid is 54 cubic inches. What is the height of the pyramid?



Warm-Up 6

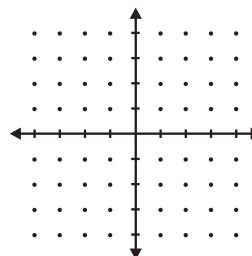
1. _____ What is the smallest positive integer that is divisible by 2, 5, 6 and 9?

2. _____ What is the value of $(0.07)^3$? Express your answer as a decimal to the nearest millionth.

3. _____ If $x = \frac{a}{b}$ and $y = \frac{b}{a}$, then what is the square of the product of x and y ?

4. _____ What is the only number that when added to its reciprocal is equal to 2?

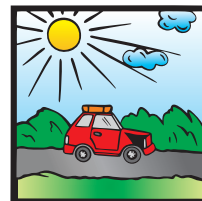
5. _____ points Two squares have centers at the origin of a coordinate plane and sides parallel to the axes. The smaller square has an area of 18 square units, and the larger square has an area of 50 square units. How many points with only integer coordinates are outside the smaller square region and inside the larger square region?



6. \$ _____ Pedro earns \$7.50 an hour at his job at Matrix Cinemas. If he works 20 hours this week and his employer withholds a total of 22% of his weekly pay for taxes and Social Security, what is his take-home pay for the week?



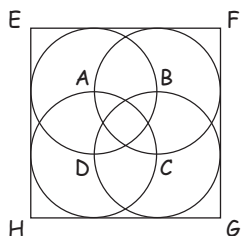
7. _____ pm Mary leaves New York City at 9:00 am, traveling to Charlotte, NC, at an average rate of 55 miles per hour. Simba leaves one hour later than Mary and follows Mary's route at an average rate of 65 miles per hour. At what time will Simba catch up to Mary?



8. (,) _____ What ordered pair of positive integers (r, s) satisfies the equation $5r + 6s = 47$, such that $r > s$?

9. _____ index cards What is the minimum number of 3-inch by 5-inch index cards needed to completely cover a 3-foot by 4-foot rectangular desktop without cutting the index cards?


10. _____ sq ft

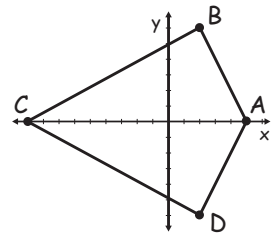


Points A, B, C and D are the centers of four circles, and they are also intersection points of these circles, as shown. Each circle has a radius of 6 feet and is tangent to two sides of square EFGH. What is the area of square EFGH?



Workout 3

- _____ When the repeating decimal $0.\overline{38}$ is expressed as a common fraction, what is the sum of the numerator and denominator?
- _____ The sum of three different two-digit prime numbers is 79. The largest of the numbers is 43. The difference of the other two is 10. What is the product of the three numbers?
- _____ Five wooden disks are numbered -3, -2, 1, 4 and 7. If two disks are chosen at random, without replacement, what is the probability that their product is negative? Express your answer as a common fraction.
- _____ The digits 1, 3, 5, 6, 7 and 9 are each used once and only once to form three two-digit prime numbers. What is the largest possible sum that can be formed?
- _____ A cylinder and a rectangular prism have the same volume. If the length, width and height of the rectangular prism are doubled and the radius of the cylinder is doubled, what is the ratio of the volume of the new cylinder to the volume of the new rectangular prism? Express your answer as a common fraction.
- (,) A quadrilateral is formed by connecting the vertices $A(5, 0)$, $B(2, 6)$, $C(-9, 0)$ and $D(2, -6)$. What would be the coordinates of point C if you moved it to the right just enough to transform quadrilateral $ABCD$ into rhombus $ABCD$? Express your answer as an ordered pair.
- _____ am Three stoplights on different streets each operate on their own independent schedules, as follows: the first stoplight is red 1 minute out of every 2 minutes (1 minute red, then 1 minute green), the second is red 2 minutes out of every 3 minutes (2 minutes red, then 1 minute green) and the third is red 3 minutes out of every 5 minutes (3 minutes red, then 2 minutes green.) At 9:00 am each stoplight turns red. The lights are either red or green (don't worry about yellow). What time is it when the next 1-minute segment of time in which all three stoplights are red begins?
- _____ If Sara makes 80% of the free throws she attempts, what is the probability that she misses exactly two of her next three free throws? Express your answer as a common fraction.
- _____ sq units  A 4-unit by 4-unit grid has this pattern shaded in the design of a company's logo. What is the area of the shaded portion of the logo?
- _____ What is the 2011th term of the arithmetic sequence $-4, -1, 2, 5, \dots$, where each term after the first is 3 more than the preceding term?





Systems of Equations Stretch

For problems 1-3, solve each of the following systems of equations. Describe the method you used. What other methods could you have used to solve each system? Express each answer as an ordered pair. Express any non-integer value as a common fraction.

1. $2x + 3y = 11$
 $3x - y = 11$

2. $\frac{2}{x} + \frac{3}{y} = 11$
 $\frac{3}{x} - \frac{1}{y} = 11$

3. $2x^2 + 3y^2 = 11$
 $3x^2 - y^2 = 11$

In problems 4 and 5, for what value of k does the linear system have no solution? Express any non-integer value as a common fraction.

4. $kx + 3y = 11$
 $3x - y = 11$

5. $2x + ky = 11$
 $3x - y = 11$

6. Given that $2a + b = 19$, $2c + d = 37$ and $b + d = 24$, what is the value of $a + b + c + d$?

7. Given that $a + b = 29$ and $ab = 204$, what is the value of $a^2 + b^2$?

8. Solve each of the following linear systems. Express your answer as an ordered pair.

a. $5x + 6y = 7$
 $8x + 9y = 10$

b. $x + 2y = 3$
 $4x + 5y = 6$

Many problems can be solved using a linear system of equations. For problems 9 and 10, use linear systems of equations to help you solve.

9a. For each equation below (i, ii, iii and iv), does there exist a solution (x, y) with positive integers x and y ? In each case, either find all pairs of integers satisfying the equation or explain why none exist. Hint for the first equation: $x^2 - y^2 = (x + y)(x - y)$, so $x + y = 12$ and $x - y = 4$ is one case to check.

i. $x^2 - y^2 = 48$

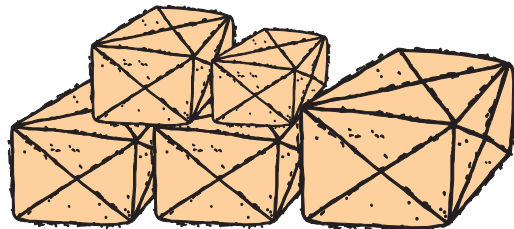
ii. $x^2 - y^2 = 23$

iii. $x^2 - y^2 = 45$

iv. $x^2 - y^2 = 90$

b. In general, for what type of integers, n , does $x^2 - y^2 = n$ have at least one solution?

10. A shipping clerk wishes to determine the weights of each of five boxes. Each box weighs a different integer amount less than 100 kg. Unfortunately the only scales available measure weights in excess of 100 kg. The clerk therefore decides to weigh the boxes in pairs so that each box is weighed with every other box. The weights for the 10 pairs of boxes are (in kilograms) 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. From this information the clerk can determine the weight of each box. What are the weights of each of the five boxes?



What About Math?

In an effort to bridge the gap between the knowledge students gain from MATHCOUNTS and the important applications of this knowledge, MATHCOUNTS is providing a *What About Math?* section in Volume I of the *MATHCOUNTS School Handbook*. This year the problems are from the fields of **engineering** and **statistics**. Written by actual professionals in these fields, these problems illustrate how students can put the skills they are learning today into practice in the near future.

This year's problems and solutions were written for MATHCOUNTS by members of the Association for Unmanned Vehicle Systems International (AUVSI), The Actuarial Foundation (TAF) and the Consumer Electronics Association (CEA).

AUVSI Foundation Writers/Editors: *Wendy Amai, Sandia National Laboratories; Dr. Kenneth Berry, Assistant Director of the Science and Engineering Education Center, University of Texas at Dallas; Angela Carr, Web Services Manager, AUVSI; Dr. Deborah A. Furey, DARPA Program Manager; Lisa Marron, Sandia National Laboratories and Dr. Dave Novick, Sandia National Laboratories*

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CEA Writers/Editors: *Bill Belt; Deepak Joseph; Brian Markwalter, PE and Dave Wilson*

ASSOCIATION FOR UNMANNED VEHICLE SYSTEMS INTERNATIONAL FOUNDATION (WWW.AUVSIFOUNDATION.ORG)

The AUVSI Foundation is a nonprofit, charitable organization that was established to support the educational initiatives of the Association for Unmanned Vehicle Systems International (AUVSI). The AUVSI Foundation's primary objective is to develop educational programs that attract and equip students for careers in robotics. To do this, the AUVSI Foundation provides K-12 and university level students with hands-on educational activities that highlight the world of robotics while emphasizing STEM curriculum—science, technology, engineering and math.

Currently, the AUVSI Foundation directly sponsors five annual advanced robotics competitions that attract nearly 200 student teams, representing more than 2000 students from schools around the world. Last year, nearly \$100,000 in prize money was awarded through these student competitions. For more information on the AUVSI Foundation and its educational programs, please visit www.auvsifoundation.org.

THE ACTUARIAL FOUNDATION (WWW.ACTUARIALFOUNDATION.ORG)

The Actuarial Foundation (TAF) was founded in 1994 as the charitable arm of the actuarial profession. TAF's mission is to develop, fund and execute education and research programs that serve the public by harnessing the talents of actuaries. Through the Foundation's youth education math activities, actuaries assist with the development of math materials that help students develop a love for math and embrace mathematical concepts in a fun and engaging way in order for them to understand how math is used in the "real world." If you like math – consider becoming an actuary. "Actuary" was rated the number two profession in the most recent *Jobs Rated Almanac*. The actuarial profession's ability to continue providing competent service in the future requires supporting mathematics achievement among today's students. The Actuarial Foundation is proud to support and sponsor MATHCOUNTS!

CONSUMER ELECTRONICS ASSOCIATION (WWW.CE.ORG)

The Consumer Electronics Association (CEA) is the preeminent trade association promoting growth in the \$165 billion U.S. consumer electronics industry. More than 2000 companies enjoy the benefits of CEA membership, including legislative advocacy, market research, technical training and education, industry promotion, standards development and the fostering of business and strategic relationships. CEA also sponsors and manages the International CES—The Global Stage for Innovation. All profits from CES are reinvested into CEA's industry services.

What About Math?

AUVSI Foundation

- 1a. _____ kg Robots often have two treads (like tank treads) instead of four wheels. You can calculate the amount of weight a tread or wheel is carrying per square inch of tread or wheel by dividing the weight of the robot by the total area of the tread or wheel on the ground.

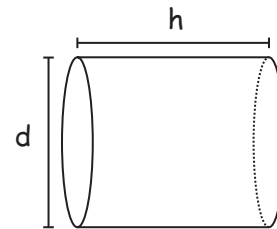
If a robot weighs 120 kg and each of the robot's 2 treads touches an area of ground that is 30 cm long and 5 cm wide, how much weight is carried over each square centimeter of tread? Express your answer as a decimal to the nearest tenth.

- 1b. _____ kg Compare this to the same robot with 4 wheels (instead of tread) that each touch a section of ground that is 3 cm \times 5 cm. How much weight is on each square centimeter of the wheels?

- 1c. _____ cm A particular robot wheel is 6 cm in diameter. The motor attached to the wheel runs at a speed of 120 rpm (rotations per minute). How many centimeters will the robot travel in 30 seconds? Express your answer as a decimal to the nearest tenth.

2. _____ kg The buoyancy of an object is defined as the mass of the displaced fluid. A particular underwater robot that has a mass of 10 kg has a volume equivalent to the right circular cylinder shown. The density of water is 1000 kg/m³, and the volume of a right circular cylinder is defined as $V = \pi r^2 h$.

For $d = 9$ cm and $h = 100$ cm (use 3.14 for π), determine if this robot will sink or float. How much flotation will need to be added (or mass will need to be added/removed) to get this robot to be neutrally buoyant in water? Express your answer as a decimal to the nearest ten thousandth. Note: For an object to be neutrally buoyant, the mass of the object must be equal to the mass of an identical volume of water.



- 3a. _____ miles An unmanned aerial vehicle (UAV) carries 45 gallons of fuel. It burns fuel at a rate of 3 gph (gallons/hour) and flies at a speed of 160 mph (miles/hour). We want to fly it to a location X, fly it in the area for 1 hour, and then fly it back to its launching point.

What is the greatest number of miles away from the UAV's launching point can location X be?

- 3b. _____ miles How far away can location X be if we want the UAV to arrive back home with a fuel reserve of 0.5 hour?

- 4a. $\frac{\text{frames}}{\text{per sec}}$ A communication network can carry 128,000,000 bits per second. A digital video camera on the network makes pictures that are 640 pixels wide by 480 pixels high. Each pixel is composed of 3 bytes. A byte is 8 bits. A picture is called a "frame."

How many frames per second from the camera can this network carry? Express your answer to the nearest whole number.

- 4b. $\frac{\text{pixels}}{\text{per sec}}$ We can increase the frame rate if we decrease the size of the picture. If we wanted to send back square frames at a rate of 30 frames/second, how many pixels wide would a square frame (same size high as wide) be? Express your answer as a decimal to the nearest tenth.

- 5a. $\frac{\text{meters}}{\text{per second}}$ Obstacles such as fallen trees, fences and ditches can be difficult for wheeled robotic vehicles to overcome. Imagine that a specially equipped robot could jump over such things. At the peak of the jump, the robot's vertical velocity is 0 m/s. Standard gravity on Earth, g , is 9.8 m/s^2 .

If the time it takes to fall from the peak is 1.7 s, how high did the robot jump? Height is equal to $\frac{1}{2}(g)(t)^2$, where g = gravity and t = time to fall to the ground, in seconds. Express your answer as a decimal to the nearest hundredth.

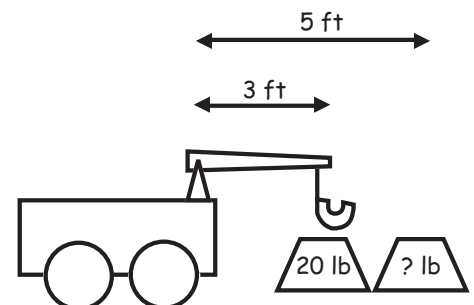
- 5b. $\frac{\text{meters}}{\text{per second}}$ How fast, in meters per second, is the robot traveling when it hits the ground? Velocity = $(g)(t)$. Express your answer as a decimal to the nearest hundredth.

6. $\frac{\text{min}}{\text{sec}}$ In 2003, NASA launched the rover Spirit onto the surface of Mars to explore our planetary neighbor. Near the end of January 2010, Mars was 99.33 million km away from Earth. Assume the speed of communication is at the speed of light, which is 300,000,000 m/s.

If a researcher on Earth requested information from Spirit about its temperature, how long did the researcher have to wait for a reply after sending the request? Express your answer in the form x minutes y seconds, where x and y are whole numbers.

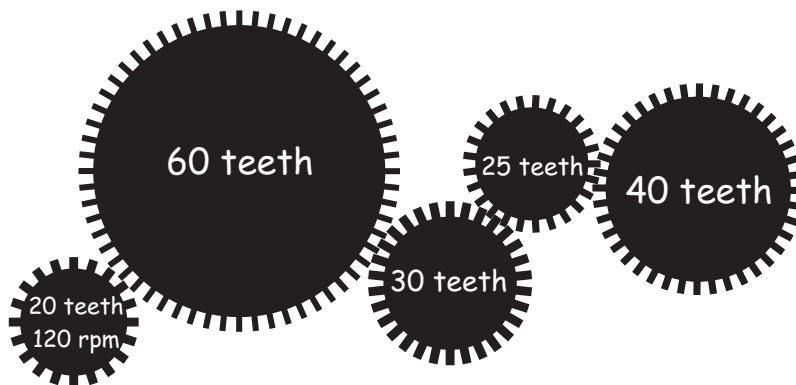
7. $\frac{\text{feet}}{\text{per min}}$ A robot has a battery life of 3 hours while carrying 0 pounds. The robot's maximum velocity is 20 feet per minute. For every pound that the robot needs to carry, the total amount of battery time the robot will operate while traveling at maximum velocity decreases by 8 minutes. Starting with a fully charged battery, how many feet can the robot travel at maximum velocity while carrying 10 pounds?

8. $\frac{\text{pounds}}{\text{per min}}$ A small ground robot has a telescoping lifting arm with a motor that can pick up a 20-pound weight when it is 3 feet away. The motor can support a maximum moment, or torque, of 60 foot-pounds (moment = weight \times distance). How much weight, in pounds, could the arm lift at 5 feet away?



9. _____ $\frac{\text{foot-}}{\text{pounds}}$ Torque is a measure of "rotational force," and it is equal to force times distance. A motor at the end of an 8-inch robot arm is lifting a 3-pound weight. How much torque is the motor generating? Note: Torque is measured in foot-pounds.

- 10a. _____ Gears perform two important duties. First, they can increase or decrease the speed of a motor. Second, they can decrease or increase the power transferred. Gears with the same size teeth are usually described not by their physical size but by the number of teeth around the circumference. The gear ratio is the relationship between the numbers of teeth on two gears that are meshed. Gear ratio = (number of teeth on output)/(number of teeth on input). When multiple gears are involved, the gear ratios from each pair of meshed gears are multiplied to calculate the gear train's gear ratio.



What is the gear ratio of the gear train shown if the rightmost gear is the input? Express your answer as a common fraction.

- 10b. _____ rpm The speed of an output gear is equal to (input speed) \times (1 \div gear ratio). If a motor with 120 rpm (revolutions per minute) is attached to the leftmost gear (input), what is the speed of the final rightmost gear that is the output?

What About Math?

The Actuarial Foundation

1. _____ The probability that a particular state will have neither a hailstorm nor a tornado in a given month is 55%. In the same period, the probability of a hailstorm is 35% and the probability of a tornado is 25%. If the probability of a hailstorm and the probability of a tornado are independent but *not mutually exclusive*, what is the probability of both a hailstorm and a tornado occurring in a given month? Express your answer as a decimal to the nearest hundredth.

Use the following information for problems 2 and 3.

The amount of damage (in millions of dollars) that could be caused by an earthquake in the fictitious country of Shakalot is shown in the table below.

Dollar Value of Damage (millions)	Probability
5	10%
10	15%
25	20%
50	30%
75	20%
100	4%
150	1%

2. \$ _____ million What is the expected (mean) damage amount, in millions of dollars, for an earthquake in Shakalot? Express your answer as a decimal to the nearest tenth.
3. \$ _____ million What is the interquartile range, in millions of dollars, for this data?
4. _____ policyholders An insurance company has sold 10,000 policies. The policyholders are classified using gender (Male or Female) and age (Young or Old). Of these policyholders, 3000 are Old, 4000 are Young Males and 4000 are Female. How many of the company's policyholders are Old Females?
5. \$ _____ Sneaky Joe has a loaded die with the numbers 1, 2 and 3 each having a probability of $\frac{1}{4}$ of being rolled and the numbers 4, 5 and 6 each having a probability of $\frac{1}{12}$ of being rolled. No one but Joe knows about the loaded die.

Sneaky Joe offers to take the following bet with you: You roll the die. If the result is an even number, you win $\$x$; if it's an odd number, you lose and pay Joe $\$5$.

How much should Sneaky Joe pay you for a win to make this a fair bet?

6. _____ % A certain disease is expected to infect 1 out of every 10,000 individuals in a country. A test for the disease is 99.5% accurate. It never gives a false indication when it is negative, so 0.5% of the people who take the test will get inaccurate readings, all of which will be false positives (meaning that the people test positive but do not have the disease).

Let us suppose you test positive; what is the probability that you actually have the disease? Express your answer as a percent to the nearest whole number.

Use the following information for problems 7-9.

Frequency = the ratio of the number of claims to the number of policies

Severity = the mean size of a group of claims

Pure Premium = $\text{Frequency} \times \text{Severity}$

Charged Premium = $\text{Pure Premium} + \text{Expenses} + \text{Profit}$

Suppose there are 50 policies that generate the following five claims in 2009:

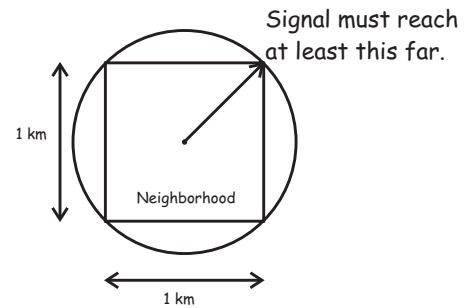
\$7500
\$5000
\$3000
\$11,500
\$8000

7. _____ $\frac{\text{claims}}{\text{policies}}$ What is the frequency? Express your answer as a decimal to the nearest hundredth.
8. \$ _____ What is the severity of the five claims?
9. \$ _____ How much premium should be charged if 15% of the charged premium is to be used for expenses and 5% of the charged premium is for profit?
10. _____ days AutoMakers, Inc. is a car manufacturer and currently has three operational plants cleverly named Plant A, Plant B and Plant C. Plant A can produce 100 cars a day. Plant B can produce 80 cars a day. Plant C can produce only 70 cars a day.
- If all three plants are running at full production capacity, how many full days are needed to produce 1600 cars?

What About Math?

Consumer Electronics Association

A provider of wireless Internet service will offer service to a neighborhood that is shaped like a square and is 1 km (1000 m) long on each side. The service provider will be using a tower that is located precisely in the middle of the square. The signal from the tower will travel an equal distance in all directions, producing a circular coverage area.



To function correctly, a consumer's wireless Internet device must receive a signal from the wireless Internet service provider that is at least 1 microwatt (1×10^{-6} watt) in strength. Radio frequency signals lose their strength as they travel over a distance according to the following formula:

Received signal strength in watts = Transmitted signal strength in watts $\div [4\pi d/\lambda]^2$, where λ is the wavelength of the wireless Internet signal in meters and d is the distance from the transmitter to the receiver in meters.

The wavelength (λ) of a radio frequency signal in meters is equivalent to the speed of light in meters per second divided by the frequency of the signal in hertz (Hz).

The speed of light is approximately 3×10^8 meters per second.

The wireless Internet service provider will be using the frequency 900 megahertz (MHz), which is equivalent to 900,000,000 Hz.

1. _____ watts How strong does the signal sent by the transmitter have to be in order to provide at least one microwatt to every receiver within the 1-square-kilometer neighborhood? Express your answer to the nearest whole number. Note: Disregard the height of the transmitters.

An electric circuit in a house goes from the circuit breaker box in the basement to the kitchen. In the kitchen there are four outlets connected to this circuit. These are the only things in this circuit.

The wire in the wall from the basement to the kitchen is able to handle 15 amps of current without becoming too hot. The circuit breaker for this circuit in the basement protects the wiring in the wall from damage. The circuit breaker will trip if more than 15 amps of current flows through the circuit.

The house has standard 120-volt electric service. In the kitchen there is one thing connected to the outlets in the circuit—an 1100-watt toaster. The power required for operating the toaster will be pulled from the total power available to the circuit. The homeowner wants to purchase a coffeemaker and connect it to another outlet in the same circuit.

2. _____ watts What is the maximum power, in watts, that the coffeemaker can consume without causing the circuit breaker in the basement to trip when both the coffeemaker and the toaster are operating simultaneously? Note: Power = voltage \times current (that is, watts = volts \times amperes).

Plug-in hybrid electric vehicles (PHEVs) combine one or more electric motors and a gas or diesel engine for propulsion. Energy is stored in a battery pack for the electric motor and in the gas tank for the combustion engine. The combination allows short trips to be made entirely on electric power and raises the effective fuel efficiency of the car. The battery in a typical PHEV stores 9000 watt-hours (W-h) of energy.

Some additional information you will need:

A house has standard 120-volt electric service.

power = voltage \times current (that is, watts = volts \times amperes)

energy = power \times time (that is, watt-hours = watts \times hours)

A standard wall outlet with a 15-ampere circuit breaker can supply 12 amperes for continuous charging, which avoids tripping the circuit breaker (circuit breakers are set 25% above the maximum continuous current a circuit is designed to carry). Electric vehicle engineers call charging from a standard outlet Level 1 charging.

3. _____ hours How long will it take to charge a PHEV with a 9000-Wh-battery from a standard wall outlet in your home? Express your answer as a decimal to the nearest hundredth.

Level 2 charging uses a 240-volt outlet that typically supplies 32 amperes of current.

4. _____ hours How long will it take to charge the same 9000-Wh-PHEV (discussed in question 3) with a Level 2 charging outlet? Express your answer as a decimal to the nearest hundredth.

Televisions use electric power while turned off and while turned on. The small amount of power used when a TV is off allows the TV to respond to a command from a remote control. Most of the power used when the TV is turned on goes toward producing the light for the picture we see on the screen. The average TV spends 5 hours a day turned on and 19 hours a day turned off. A 55-inch TV uses 197 watts/hour when it is on and 0.4 watt/hour when it is off. Thus, a 55-inch TV uses 992.6 watts per day, which can be calculated by adding together the power used while the TV is on and the power used while the TV is off each day. The power used while the TV is on is found by multiplying the number of hours the TV is on times the power used per hour while it is on. A corresponding formula applies for the power used while the TV is off.

5. _____ watts How many watts per day does a small TV use if it uses 33 watts/hour while on and 0.4 watt/hour while off? Assume that the TV is on for 5 hours and off for 19 hours. Express your answer as a decimal to the nearest tenth.

Digital displays (such as TVs and computer monitors) are built according to standard dimensions (width and height) and resolutions (pixels). The width and height dimensions of a display can also be expressed as a ratio called the aspect ratio. The aspect ratio of a display is the ratio of the width of the image to its height, expressed as two numbers separated by a colon. The resolution of a display is the number of distinct pixels (smallest elements of a screen) that can be displayed in each dimension.

For example, a display that has resolution of 800×600 pixels, or 800 pixels in the width dimension and 600 pixels in the height dimension has an aspect ratio of 4:3. This is because the aspect ratio 800:600 can be reduced to 4:3.

6. _____ If a high-definition digital TV has a resolution of 1920×1080 pixels, what is its aspect ratio? Express your answer in the form $a:b$, where a and b are positive integers and have no common factors other than 1.

Internet speed to a home is 25 megabits per second (Mbps) peak throughput. Viewing or using certain common websites takes up varying bandwidth as shown in the list below:

- (a) Viewing MySpace takes up 1 Mbps
- (b) Using Pandora takes up 5 Mbps
- (c) Viewing Facebook takes up 2 Mbps
- (d) Viewing Amazon On Demand takes up 24 Mbps
- (e) Viewing YouTube-HD takes up 12 Mbps

For example: Viewing Myspace (1 Mbps) and listening to Pandora (5 Mbps) at the same time uses $1 \text{ Mbps} + 5 \text{ Mbps} = 6 \text{ Mbps}$ of bandwidth of the 25 Mbps available. This is because bandwidth usage is additive. If a group of websites exceeds the available bandwidth, they cannot all be used simultaneously.

7. _____ websites What is the maximum number of distinct websites from the above list that can be simultaneously used at the home?

